3 OPERATEURS VECTORIELS DIFFERENTIELS

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Licence et masters

1 Cartésiens

$\vec{grad}$ 0.1 $$\nabla g = \frac{\partial g}{\partial x}\vec{u}_x + \frac{\partial g}{\partial y}\vec{u}_y + \frac{\partial g}{\partial z}\vec{u}_z = \vec{grad}\, g$$

div 0.2 $$\nabla \vec{G} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z} = \mathrm{div}\,\vec{G}$$

$\vec{rot}$ 0.3 $$\vec{\nabla} \wedge \vec{G} = \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}\right)\vec{u}_x + \left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}\right)\vec{u}_y + \left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right)\vec{u}_z = \vec{rot}\,\vec{G}$$

Δ 0.4 $$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2} = \Delta g$$

2 Cylindriques

0.5 $$\nabla g = \frac{\partial g}{\partial \rho}\vec{u}_\rho + \frac{1}{\rho}\frac{\partial g}{\partial \varphi}\vec{u}_\varphi + \frac{\partial g}{\partial z}\vec{u}_z$$

0.6 $$\nabla \vec{G} = \frac{1}{\rho}\frac{\partial(\rho G_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial G_\varphi}{\partial \varphi} + \frac{\partial Gz}{\partial z}$$

0.7 $$\vec{\nabla} \wedge \vec{G} = \left(\frac{1}{\rho}\frac{\partial G_z}{\partial \varphi} - \frac{\partial G_\varphi}{\partial z}\right)\vec{u}_\rho + \left(\frac{\partial G_\rho}{\partial z} - \frac{\partial G_z}{\partial \rho}\right)\vec{u}_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho G_\varphi)}{\partial \rho} - \frac{\partial G_\rho}{\partial \varphi}\right)\vec{u}_z$$

0.8 $$\nabla^2 g = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial g}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 g}{\partial \varphi^2} + \frac{\partial^2 g}{\partial z^2}$$

3 Sphériques

0.9 $$\nabla g = \frac{\partial g}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial g}{\partial \theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial g}{\partial \varphi}\vec{u}_\varphi$$

0.10 $$\nabla \vec{G} = \frac{1}{r^2}\frac{\partial(r^2 G_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(G_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial G\varphi}{\partial \varphi}$$

0.11 $$\vec{\nabla} \wedge \vec{G} = \frac{1}{r\sin\theta}\left(\frac{\partial(G_\varphi \sin\theta)}{\partial \theta} - \frac{\partial G_\theta}{\partial \varphi}\right)\vec{u}_r$$
$$+\left(\frac{1}{r}\frac{1}{\sin\theta}\frac{\partial G_r}{\partial \varphi} - \frac{1}{r}\frac{\partial(r\, G_\varphi)}{\partial r}\right)\vec{u}_\theta$$
$$+\frac{1}{r}\left(\frac{\partial(rG_\theta)}{\partial r} - \frac{\partial G_r}{\partial \theta}\right)\vec{u}_\varphi$$

0.12 $$\nabla^2 g = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial g}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial g}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 g}{\partial \varphi^2}$$

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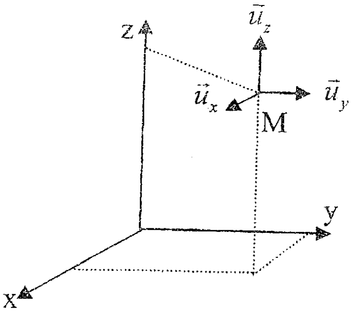
1 LES SYTEMES de COORDONNEES

1. Cartésiennes

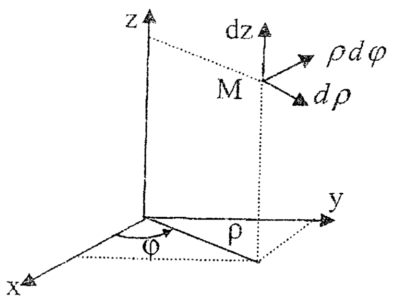


M(x, y, z)

$$M\begin{vmatrix} x \\ y \\ z \end{vmatrix} \qquad \overrightarrow{dS}\begin{vmatrix} \overrightarrow{dx} = dx\,\vec{u}_x \\ \overrightarrow{dy} = dy\,\vec{u}_y \\ \overrightarrow{dz} = dz\,\vec{u}_z \end{vmatrix}$$

trièdre $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$ *direct*

2. Cylindriques

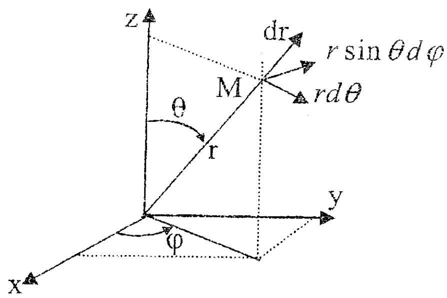


M(ρ, φ, z)

$$M\begin{vmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ z = z \end{vmatrix} \qquad \overrightarrow{dS}\begin{vmatrix} \overrightarrow{d\rho} = d\rho\,\vec{u}_\rho \\ \rho\overrightarrow{d\varphi} = d\varphi\,\vec{u}_\varphi \\ \overrightarrow{dz} = dz\,\vec{u}_z \end{vmatrix}$$

trièdre $(\vec{u}_\rho, \vec{u}_\varphi, \vec{u}_z)$ *direct*

3. Sphériques



M(r, θ, φ)

$$M\begin{vmatrix} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{vmatrix} \qquad \overrightarrow{dS}\begin{vmatrix} r\overrightarrow{d\theta} = d\theta\,\vec{u}_\theta \\ r\sin\theta\overrightarrow{d\varphi} = d\varphi\,\vec{u}_\varphi \\ \overrightarrow{dr} = dr\,\vec{u}_r \end{vmatrix}$$

trièdre $(\vec{u}_\theta, \vec{u}_\varphi, \vec{u}_r)$ *direct*

$$\begin{vmatrix} \vec{u}_x = \vec{u}_\rho\cos\varphi - \vec{u}_\varphi\sin\varphi & \text{C1} \\ \vec{u}_y = \vec{u}_\rho\sin\varphi + \vec{u}_\varphi\cos\varphi & \text{C2} \\ \vec{u}_z = \vec{u}_z & \text{C3} \end{vmatrix}$$

$$\begin{vmatrix} \vec{u}_x = \vec{u}_r\sin\theta\cos\varphi + \vec{u}_\theta\cos\theta\cos\varphi - \vec{u}_\varphi\sin\varphi & \text{C7} \\ \vec{u}_y = \vec{u}_r\sin\theta\sin\varphi + \vec{u}_\theta\cos\theta\sin\varphi + \vec{u}_\varphi\cos\varphi & \text{C8} \\ \vec{u}_z = \vec{u}_r\cos\theta - \vec{u}_\theta\sin\theta & \text{C9} \end{vmatrix}$$

$$\begin{vmatrix} \vec{u}_\rho = \vec{u}_x\cos\varphi + \vec{u}_y\sin\varphi & \text{C4} \\ \vec{u}_\varphi = -\vec{u}_x\sin\varphi + \vec{u}_y\cos\varphi & \text{C5} \\ \vec{u}_z = \vec{u}_z & \text{C6} \end{vmatrix}$$

$$\begin{vmatrix} \vec{u}_r = \vec{u}_x\sin\theta\cos\varphi + \vec{u}_y\sin\theta\sin\varphi + \vec{u}_z\cos\theta & \text{C10} \\ \vec{u}_\theta = \vec{u}_x\cos\theta\cos\varphi + \vec{u}_y\cos\theta\sin\varphi - \vec{u}_z\sin\theta & \text{C11} \\ \vec{u}_\varphi = -\vec{u}_x\sin\varphi + \vec{u}_y\cos\varphi & \text{C12} \end{vmatrix}$$

2 IDENTITES VECTORIELLES

Vecteurs $\vec{A}, \vec{B}, \vec{C}, \vec{G}, \vec{F}, \vec{H}$

Scalaires f, g

$\nabla g = \overrightarrow{grad}\ g = \vec{\nabla} g$

$\nabla\vec{G} = div\ \vec{G} = \vec{\nabla}\vec{G}$

$\vec{\nabla}\wedge\vec{G} = \overrightarrow{rot}\ \vec{G} = \nabla\times\vec{G}$

I - 1 $\quad \vec{A}\wedge(\vec{B}\wedge\vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$

I - 2 $\quad (\vec{A}\wedge\vec{B})\wedge\vec{C} = (\vec{C}.\vec{A})\vec{B} - (\vec{C}.\vec{B})\vec{A}$

I - 3 $\quad \nabla.(\vec{\nabla}\wedge\vec{G}) = 0$

I - 4 $\quad \vec{\nabla}\wedge\nabla g = 0$

I - 5 $\quad \nabla.\nabla g = \nabla^2 g = \Delta g$

I - 6 $\quad \nabla(f+g) = \nabla f + \nabla g$

I - 7 $\quad \nabla(\vec{F}+\vec{G}) = \nabla.\vec{F} + \nabla.\vec{G}$

I - 8 $\quad \nabla(fg) = g\nabla f + f\nabla g$

I - 9 $\quad \nabla(f\,\vec{G}) = \vec{G}(\nabla.f) + f(\nabla.\vec{G})$

I - 10 $\quad \vec{\nabla}\wedge(f\,\vec{G}) = (\nabla f)\wedge\vec{G} + f(\vec{\nabla}\wedge\vec{G})$

I - 11 $\quad \vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{G}) = \nabla(\nabla.\vec{G}) - \nabla^2\vec{G}$ ← rot rot $\vec{G}$ = grad div $\vec{G}$ − $\Delta\vec{G}$

I - 12 $\quad \nabla^2\vec{G} = \vec{u}_x\nabla^2 G_x + \vec{u}_y\nabla^2 G_y + \vec{u}_z\nabla^2 G_z$

I - 13 $\quad \vec{\nabla}.(\vec{F}\wedge\vec{G}) = \vec{G}.(\vec{\nabla}\wedge\vec{F}) - \vec{F}.(\vec{\nabla}\wedge\vec{G})$

I - 14 $\quad \vec{F}.(\vec{G}\wedge\vec{H}) = \vec{G}.(\vec{H}\wedge\vec{F}) - \vec{H}.(\vec{F}\wedge\vec{G})$

I - 15 $\quad \vec{\nabla}\wedge(\vec{F}\wedge\vec{G}) = \vec{F}.(\nabla.\vec{G}) - \vec{G}.(\nabla.\vec{F}) + (\vec{G}.\vec{\nabla})\vec{F} - (\vec{F}.\vec{\nabla})\vec{G}$

I -16 $\quad \vec{\nabla}.(\vec{F}.\vec{G}) = (\vec{F}.\vec{\nabla})\vec{G} + (\vec{G}.\vec{\nabla})\vec{F} + \vec{F}\wedge(\vec{\nabla}\wedge\vec{G}) + \vec{G}\wedge(\vec{\nabla}\wedge\vec{F})$

I - 17 $\quad \iiint_v \nabla.\vec{G}\ dv = \iint_S \vec{G}.\vec{n}\ dS$ théorème de la divergence v fermé par S

I - 18 $\quad \iint_S (\vec{\nabla}\wedge\vec{G}).\,\vec{n}\ dS = \oint_\ell \vec{G}.\vec{d\ell}$ théorème de Stokes contour ℓ de la surface S